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An epidemic spreading model based on community structure in dual social networks

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Abstract

The spread of disease through a physical contact network and the diffusion of information awareness about the disease on a communication network are two intimately related dynamical processes. To catch the influence of community structure on the epidemic spreading, we propose an epidemic spreading model based on the network community structure in dual networks. The model used the dual network to describe the interplay between the two types of spreading dynamics, each occurring on its own layer. As for the diffusion of awareness, we study the impact of the community structure on the changes of awareness diffusion process which will alter the epidemic threshold and the final infected size. Through Markov chain approach and numerical computation, we derive the epidemic threshold of the model. Theoretical analysis shows that the existence of the community structure in the consciousness layer network increases the awareness probability in the network, and then increases the epidemic threshold. Simulation results indicate that the epidemic threshold of the dual network propagation model based on community structure is higher, and the infected size is lower than that without community structure when the system is in balance.

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Introduction

As the diffuse of information awareness about the disease will affect the spread of epidemic, exploring the interplay between disease information awareness diffusion and epidemic spreading is a topic that has been receiving increasing attention. To describe the co-evolution of two processes, one of the most important methods, multiplex networks model, is carried out.

Through embodying the transmission process of the infectious disease and information awareness about the disease, the multiplex networks epidemic spreading models which are aimed at different awareness diffusion process were proposed (Granell, 2013, or Guo, 2015).

A pioneering step in this direction was taken by Granell who studied the epidemic spreading in the multiplex networks by establishing two layers network, in which one represents epidemic spreading and another represents the diffusion of the information awareness. And the results shown that there is a critical point at which the disease spreading can be controlled by the individual information awareness (Granell *et al*., 2013).

Except for the above article, Granell also considered the importance of media coverage for the diffusion of the information awareness, and the simulation results showed that the media broadcast will make the critical point disappear (Clara *et al*., 2014). Wang studied the asymmetric coupling between the diffusion of the disease information awareness and the disease, and the results showed that the epidemic spreading on the contagion network can induce the diffusion of the disease awareness, on the contrary, the diffusion of the awareness can inherent the disease spreading (Wang *et al*., 2014).

Under the framework of the multiplex networks, there has been growing interesting in exploring the topology effects of the multiplex networks on the dynamic processes.

In view of this problem, Guo used the activity driven machine to modeling the change of the awareness network topology and gave out a model which name is epidemic spreading with activity-driven awareness diffusion on the multiplex network. The results showed that small changes in the network topology of the information communication layer will directly affect the value of critical point infectious disease (Guo *et al*., 2016).

Exploring the topology of the network plays a big role in understanding the epidemic spreading process in the network. As one of most important topologies for the network, community structure which widely exists in social networks has received a great amount attention (Newman., 2012) and a large number of scholars have studied the impact of community structure on the epidemic spreading from the aspects of the degree distribution, the degree of overlap, the size of the community and so on (Peng, 2013, Shang, 2015, Stegehuis, 2016, or Kashisaz *et al*., 2016).

For example, Pen constructed two community networks with different degree distribution and found that the communities with large degree distribution are more easily staying in the infected state, however, the communities with small degree distribution will stay in the situation in which the outbreak and disappearance of the infectious disease will happen alternately (Peng *et al*. 2013).

The study proposed by Shang put forward that how the overlap area in the community structure influences the spread of infectious diseases, and the results showed that the higher degree of overlap area has, the faster the disease spread in the network. In addition, the results showed that the average degree of the network is important to the epidemic spreading (Shang *et al*. 2015).

Kashisaz implemented the parameters of the community structure size and the mixing parameter between different communities into account, used the SI model to describe the epidemic spreading, the results showed that increasing the size of community will reduce the incidence of the infectious disease, and increasing the interaction between communities will increase the incidence of disease (Kashisaz *et al*., 2016). These papers discussed above have described the influence on the epidemic spreading induced by the community structure in different network, however, the paper are focused on the single-layer network. Once an infectious disease spreading, the awareness individuals tend to take some protected measures, which causing the diffusion of the information awareness about the disease on their community.

The individuals in different communities will have different awareness getting probabilities. Consequently, catching the features of the epidemic spreading model in the dual networks with considering the community structure is important.

Compared with previous studies, an epidemic spreading model based on the community structure in the dual network is proposed. The model considers that the impact of the community structure on the awareness diffusion, and analysis the awareness getting probability changes in the spreading of infectious disease. Firstly, we use probability tree to indicate the state transform probability that belongs to each individual in different time and obtain the epidemic threshold of the model by the Markov chain.

Furthermore, we numerically study the effect of community structure on individual's self-protection and the epidemic threshold. Finally, we study the interplay between the two dynamic processes in our model by discussing the changes of the epidemic threshold and the infected size induced by altering the infected probability and the awareness probability.

Materials and methods

Model

When a disease emerges, infectious diseases will spread in the human contact network. During the epidemic spreading, the individuals will obtain the infectious disease information through the television or the network, then will obtain the infectious disease consciousness. In other words, the infectious disease spreads along with the infectious disease awareness dissemination in the social network. In view of the above phenomena, a dual network was established to represent the interaction between the transmission of infectious diseases and the spread of disease information awareness (Granell *et al*., 2013).

In the established dual network, a sub-network represents the disease information awareness diffusion network and another sub-network represents the infectious disease spreading network. Since the spreading of the disease is limited by the geographical position, while, with help of advance in the communication network, the diffusion of the disease information awareness will not limited by it.

Consequently, the link of the network in which the disease spreads will less than that in the awareness network. Therefore, the model we proposed uses the graph $G_1(V_1, V_2)$and $G_2(V_1, V_2)$ to represent the awareness layer network and the contagion layer network, respectively, where $V_1$and V_2 are the sets of the individuals in two layers, E_1 and $E_2$are the link sets of the node in the awareness layer and the contagion layer. And we consider each network in the dual network including$N$ nodes, and we assume $|E_1| < |E_2|$and$|V_1| = |V_2| = N$.

In this paper, we will discuss the impact of the community structure on the awareness diffusion process. Therefore, we divided the awareness network into different communities. As shown in the Fig. 1, the awareness network is divided into three different communities: community one, community two and community three. In addition, the edge between the nodes with the same community is dense while that between the different communities is sparse.

In Fig. 1, we should note that, in awareness network, the white node represents the node stay in the awareness state (A)and the gray node represents the node stay in the unawareness state (U). Moreover, in the contagion network, the white node represents the node is stay in the susceptible state (S) and the gray node represents the node stays in the infected state (I).

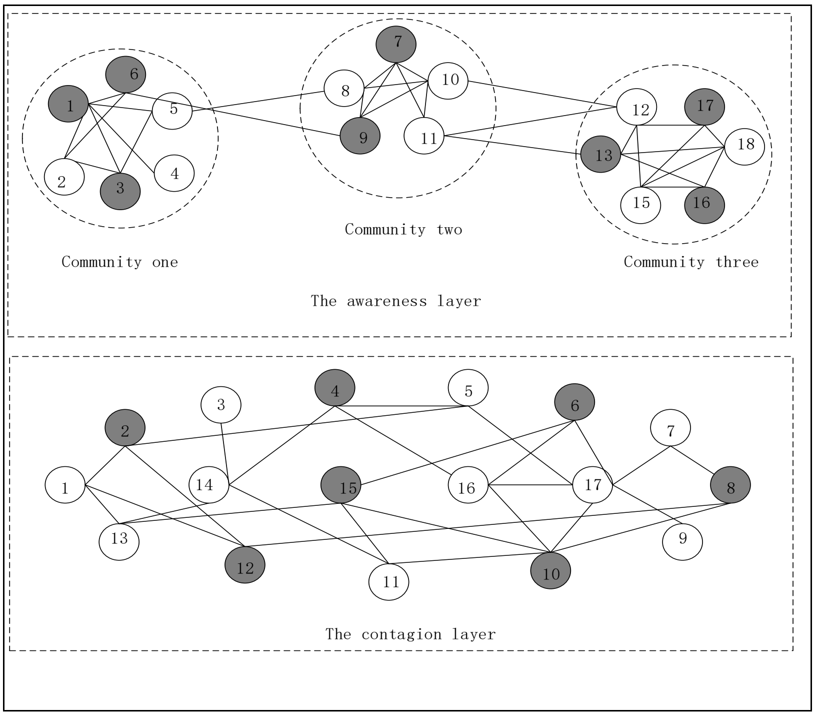


Fig. 1. The dual network model.

Description: The model of dual networks based on the community structure. The networks are different per layer, and each node in one layer is connected to its counterpart in the other layer. According to the above assumption, there are three kinds of individual state in the dual networks: conscious of the infected (AI), unconscious susceptible (US), conscious susceptible (AS).

The evolution of the node state

In the awareness network, the conscious node will forget the awareness about the infectious disease with probabilityδ, and will become unconscious node. While the unawareness node will obtain the awareness with probability λ. Since the relationship between the individuals in the same community is stronger than that in different community, and the communication frequency between the same community individuals is frequent than that in different community, the trust degree when individual exchange their disease information will increase in the same community and the individual awareness probability will be increased. Consequently, our model proposed a reliability parameter $\theta(C_i, C_j)$ to adjust the awareness degree for the infectious disease during the community. Moreover, the value of the parameter in the same community is larger than that in different communities. In the contagion layer, the infected node will become susceptible node with probabilityβ, and the susceptible node will infect the disease with probabilityα. Since the infected probabilities belong to the awareness node and the unawareness node are different, we use α^A and α^U to represent their infected probabilities.

In order to analyze the coupled dynamical processes in the dual networks, we need to explore the details of the evolution of the individual states. Firstly, we will initiate individual states. In the infectious disease network, we randomly select$m$ nodes as infected nodes, the rest nodes are in the susceptible state. In the level of consciousness network, the node corresponding to the node in the infected state is conscious nodes. and the rest nodes who corresponding state in the awareness network are unawareness stay in the susceptible state.

After the initialization of the individual states, the evolution of the node states in the dual network based on the community structure follows two rules: R1 (the rule of disease infection and consciousness getting) and R2 (the rule of disease recovery and consciousness loss). The two rules are described as follows:

R1: After the selection of the infected and awareness node$i$, node j will get the awareness of the disease with probability $\theta(C_i, C_j) \cdot \lambda$by communicating with node$i$. If the node j becomes aware of the disease, the node will infect the disease with probability α^A, otherwise, it will infect the disease with probability α^U in the contagion network.

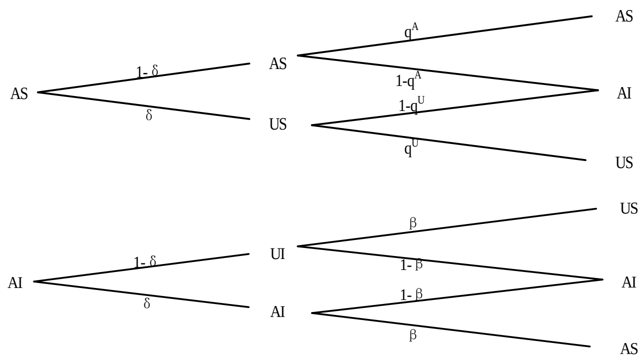
R2: In the awareness network, the conscious node forgets the infectious disease consciousness with probabilityδ and it will become unconscious node. In the contagion network, the infected node becomes a susceptible node with probabilityβ.

According to the above rules, the spread of infectious diseases causes the spread of diseases awareness in the consciousness layer network and the diffusion of the awareness influences the epidemic spreading.

The two sub-networks interact with each other, and the individual state in the dual network evolves alternatively.

Methods

The probability trees which are used to describe the individual state in different time and the transition probability between the different states is given in the Fig. 2. Here, let a_{ij}, b_{ij}be the adjacency matrices of the awareness layer and the contagion layer, respectively. Since the individual i has to be one of the three states at time t, we denote the probabilities as $p_i^{AI}(t)$, $p_i^{AS}(t)$,$p_i^{US}(t)$, respectively. Then on the conscious layer, we define the probability for unconscious individual i not changing from unawareness state to awareness state as $r_i(t)$. On the contagion layer, we define the probabilities for individual being infected by any neighbors if $i$was aware as $q_i^A(t)$, and not being infected by any neighbors if $i$was unaware as $q_i^U(t)$.



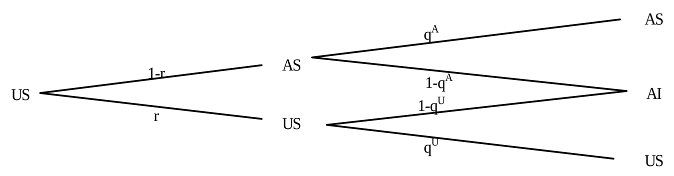


Fig. 2. Transition probability trees for the three possible node states.

Description: The possible node states are AI, US, and AS. Note thatβ represents the probability of the transition from infected to susceptible, δ represents the probability of the transition from aware to unaware.

According to the define of $r_i(t)$, we can see that the value is related to whether the neighbor nodes of node i stay in awareness. Given the consideration about the community structure influence on the awareness probability, if node i and node j belong to the same community, the information trust degree will increase causing the awareness getting probability increasing, so that the value of λ and $r_i(t)$ will change.

According to the above definition, we get

$$r_i(t) = \prod_{j=1}^{N}(1 - a_{ji} \cdot p_j^A(t) \cdot \theta(C_i, C_j) \cdot \lambda\ ),$$
$$q_i^A(t) = \prod_{j=1}^{N}(1 - b_{ji}p_j^{AI}(t)\alpha^A), \qquad (3)$$
$$q_i^U(t) = \prod_{j=1}^{N}(1 - b_{ji}p_j^{AI}(t)\alpha^U).$$

Combining the probability tree and Markov chain gives that

$$p_i^{US}(t+1) = p_i^{US}(t)r_i(t)q_i^U(t) + p_i^{AS}(t)\delta q_i^U(t) + \delta\beta p_i^{AI}(t),$$
$$p_i^{AS}(t+1) = p_i^{US}(t)\ (1 - r_i(t))\ q_i^A(t) + p_i^{AS}(t)(1 - \delta)q_i^A(t) + p_i^{AI}(t)\beta(1 - \delta, \qquad (4)$$
$$p_i^{AI}(t+1) = p_i^{US}(t)\{[1 - r_i(t)][1 - q_i^A(t)] + r_i(t)[1 - q_i^U(t)]\} + p_i^{AI}(t)(1 - \beta)$$
$$+p_i^{AS}(t)\{(1 - \delta)\left([1 - q_i^A(t)\right) + \delta(1 - q_i^U(t))\}.$$

It is necessary to find the stationary solution of the above equation in order to calculate the epidemic threshold, When time $t \to \infty$, there exists an epidemic threshold α_c^U for the coupled processes, which means the epidemic can outbreak only if $\alpha > \alpha_c^U$. By letting $t \to \infty$, the probabilities of three states p_i^{AI},p_i^{AS},p_i^{US}fulfill the condition that

$$\lim_{t\to\infty} p_i^{US}(t+1) = p_i^{US}(t+1) = p_i^{US},$$
$$\lim_{t\to\infty} p_i^{AS}(t+1) = p_i^{AS}(t+1) = p_i^{AS}, \qquad (5)$$
$$\lim_{t\to\infty} p_i^{AI}(t+1) = p_i^{AI}(t+1) = p_i^{AI}.$$

Since around the epidemic threshold α_c^U, the infected probability $p_i^{AI} = \epsilon_i \ll 1$, the probabilities $q_i^A$and q_i^U can be simplified as

$$q_i^A \approx\ (1 - \alpha^A \sum_j b_{ji}\,\epsilon_j)\ , \qquad (6)$$
$$q_i^U \approx\ (1 - \alpha^U \sum_j b_{ji}\,\epsilon_j)\ .$$

Therefore, inserting these approximations into Eq. (3) and omitting higher order items, Eq. (3) is reduced to the following form.

$$\varepsilon_i\beta = p_i^{Us}\alpha^U \sum_j\ b_{ji}\varepsilon_j. \qquad (7)$$

It is clear that $p_i^{US} + p_i^{AS} + = 1$, where $p_i^A = p_i^{AS} + p_i^{AU}$. Noting that $p_i^{AI} = \epsilon_i \ll 1$, we get $p_i^A \approx p_i^{AS}$and $p_i^{US} = 1 - \left(p_i^{AS} + p_i^{AI}\right) = 1 - p_i^A$. Hence

$$\sum_j\ [\ (1 - p_i^A)\ b_{ji} - \frac{\beta}{\alpha^U} t_{ji}]\varepsilon_j = 0. \qquad (8)$$

Where$h_{ij}$are the elements of the identity matrix. Let H be a matrix whose element h_{ij} equals to $(1 - p_i^A)\,b_{ji}$. Then, it is obvious that the Eq. (8) has nontrivial solutions if and only if $\frac{\beta}{\alpha^U}$ is the eigenvalue of matrix H. Consequently, the epidemic threshold α_c is the one which satisfies $\Lambda_{max} = \frac{\beta}{\alpha^U}$, where Λ_{max} is the largest eigenvalue of matrix H, and it is easy to get

$$\alpha_c^U = \frac{\beta}{\Lambda_{max}}. \qquad (9)$$

Note that α_c depends explicitly on the dynamics on the contagion layer, in particular of the value of p_i^A. Interestingly, if we consider the critical value $\lambda_c = {}^{\delta}/_{\Lambda_{max}(A)}$ of the onset of awareness without considering the spreading of the infection, then for $\lambda < \lambda_c$ Eq. (9) reduces to $\alpha_c = {}^{\beta}/_{\Lambda_{max}(B)}$, and the Onset of the disease, is obviously independent of the awareness. However, for values of $\lambda > \lambda_c$ the epidemic threshold is related to the recovery probability $\beta$and the topology of the awareness. Specifically, it depends on the variety values of p_i^A, influenced by the community structure of the awareness layer.

Results

In this section, we will analyze the model from two different parts. In the first part, we will explore the influence of the community structure. Then, the interaction between the two spreading processes will be discussed in the second part.

To clearly present our results, we first describe our two-layer multiplex network model of epidemic spreading considering the community structure.

(1) A dual network is constructed according to the BA network generating algorithm (Barabasi *et al*., 1999).

(a) The bottom layer corresponding to the physical contact network is free-scale network with an exponent of 2.5 and a size of 1000 nodes.

(b) The top layer representing the information contacts is the same network with 400 additional (non-overlapping with previous) links.

(2) The dual network is divided into disjoint communities according to the community structure finding algorithm CNM proposed by Newman (Cluster *et al*., 2005).

Some infected nodes is selecting from the generated network. All the simulations start from a fraction ρ_0 of randomly chosen infected nodes and ρ_0 is fixed to be 0.2. Iterate the rules that described in the Section 4 of the coupled dynamical.

Processes with parallel updating until the density of the infected nodes ρ^I is steady. In order to reduce the fluctuation of the density ρ^I, we make time average that satisfies $\rho^I = \frac{1}{T}\sum_{t=t_0}^{t=t_0+T-1} \rho^I(t)$ and take $T = 100$ $(t_0 = 901)$.

The influence of the community structure on the epidemic spreading

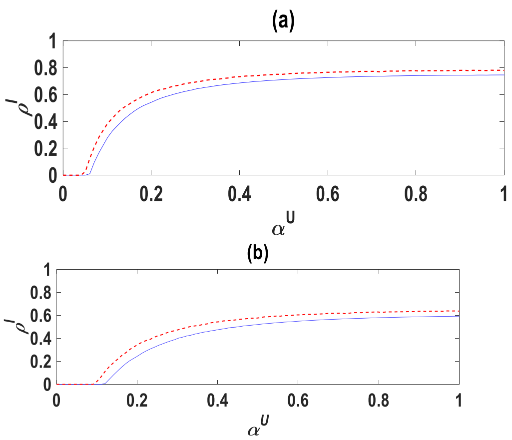
In this section, we will explore the community structure efficiency on the dual network epidemic spreading process. In Fig. 3, we show the comparison between our model and the model proposed by Granell (Granell *et ai.,* 2013).

With different disease infected probability. Here, let ρ^Irepresents the fraction of the infected node and α^U represent the infected probability belongs to the unawareness node.

As shown in Fig. 3, we assume the value of the probability of getting infectious disease is$(a)\alpha = 0.2, (b)\alpha = 0.4, (c)\alpha = 0.6, (d)\alpha = 0.8,$ and the recovery probability and the awareness forgetting probabilityare 0.2 and 0.8, respectively.

It is assumed that the awareness getting probability of individuals between different communities is the same as that in the network without considering the community structure, and the awareness probability of the individuals who belong to the same community is higher than that in the network without considering the community.

For simplicity, we assume that the value of the trust parameter the individual within the same community have is $\theta(C_i, C_j) = 2$, and that the individual in different communities is $\theta(C_i, C_j) = 1$.



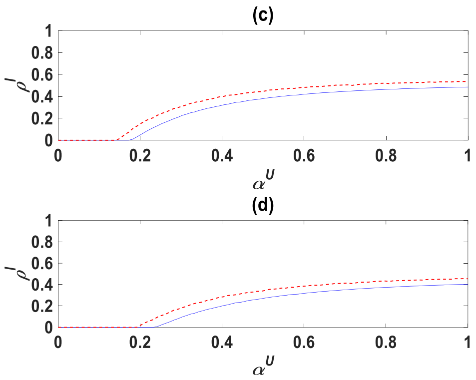


Fig. 3. Community Efficient on the epidemic spreading.

Description: The comparisons of the epidemic threshold between our model and the model without considering the community structure.

In Fig. 3, the dotted line is drew the fraction of the infected node in the model without considering the community structure, and the solid line is drew the fraction changes with the different infected probability in our model. According to the Fig. 3, we can observe some questions.

As far as the epidemic threshold, the epidemic threshold in the solid line is larger than that in the dotted line. For example, in Fig. 3(c), the epidemic threshold value of the dotted line is$\alpha^U = 0.148$, however, the value in the solid line is $\alpha^U = 0.179$. In view of the size of the epidemic spreading, in the solid model, it is smaller than that in the dotted model. For example, in Fig. 3(c), the value of the epidemic size in the solid line is 0.52, and that in the dotted line is 0.57. As far as the gap between the two models, with the increasing of the infected probability, the gap between the two models is increasing. For example, when the infected probability is 0.2, the value of the gap is 0.015, while when the infected probability is 0.4, the value of the gap is 0.03. In view of the finally epidemic size gap between the two models, there are no obvious changes with the increasing of the infected probability.

The analysis given above shows that the existence of the community structure let the individual in the same group get the awareness easily, then the awareness getting probability in the network is increasing, so that the epidemic threshold increases and the epidemic size becomes small.

The analysis of the interplay between the dual networks

In this part, we will discuss the interaction between the two diffusion processes. For the proposed model, we plot the color chart by changing the value of the infected probability belongs to the unawareness individual and the awareness obtaining probability. We assume that $(a)\beta = 0.2, \delta = 0.8, (b)\beta = 0.4, \delta = 0.6, (c)\beta = 0.6, \delta = 0.4, (d)\beta = 0.8, \delta = 0.2$.

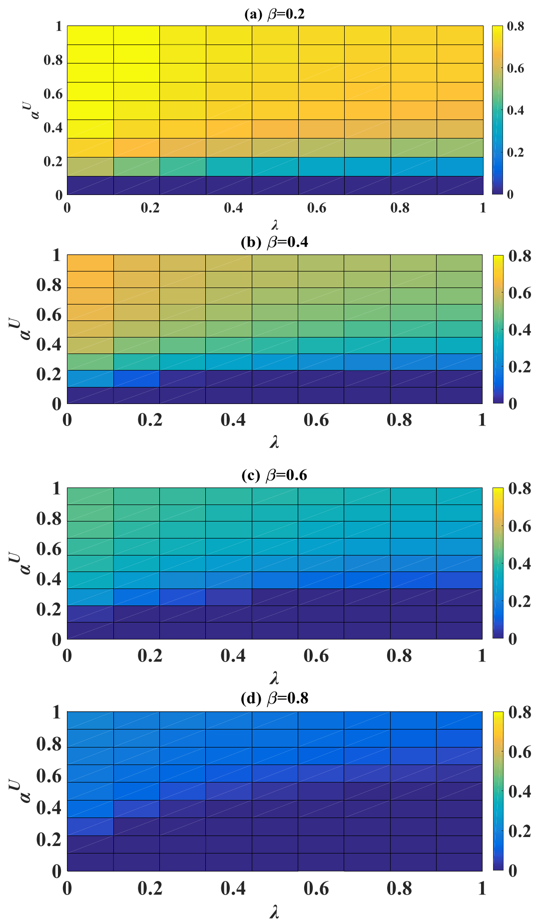


Fig. 4. Interaction between the dual networks.

Description: The simulation results of the infected node size as a function of the infected probability and the awareness getting probability.

In Fig. 4, the intensity of the color represents the density of the infected individual. The darker the color is, the lower the infected individual density in the network is, on the contrary, the lighter the color is, the higher the infected individual density is. The value of the unawareness individual infected probability is represented by α^{U}, we use λ to represent the awareness getting probability. When we fix the value of the awareness getting probability, with the same value of awareness forgiving probability and the disease recovery probability, the color of the panel is becoming light when the infected probability increases. In other words, the fraction of the infected node is increasing. In the Fig. 4(a), we assume that $\beta = 0.2, \delta = 0.8$. When we fix the value $\beta = 0.2, \delta = 0.8$, the color of the picture is becoming light. On the contrary, when we fix the value of the infected probability, with the increasing of the value of the awareness getting probability, the color is becoming dark which indicates the fraction of the infected node decreases. In the Fig. 4(c), we assume that $\beta = 0.6, \delta = 0.4$, when we fix the value$\alpha^{U} = 0.2$, the color of the Fig. is becoming darker. According to the comparison with the Fig.4 (a),(b),(c) and (d), we can observe that the color of the Fig. changes obviously when the recovery probability increases and the awareness forgiving probability decreases.

The analysis shows that the influence on the epidemic spreading induced by the community structure of the awareness network will become small when the forgetting probability increases and the recovery probability decreases, and the epidemic will outbreak more easily. Therefore, in the process of prevention and control of infectious diseases, we can delay the outbreak of infectious diseases by raising the awareness of infectious diseases.

Discussion

To summarize, we have considered the community structure of the awareness layer network, and discussed the influence of community structure on the epidemic spreading. The two-layer network is established, and the awareness network is divided into different communities. In addition, we analysis the changes of the awareness diffusion process in the dual network by considering the community structure, and the probability of the state transformation at different time is expressed by the probability trees.

The epidemic threshold of the model is obtained by using the Markov chain to list the equation according to the transformed probability. The theoretical analysis shows that the epidemic threshold of infectious diseases is indeed affected by the transmission of the awareness which influenced by the community structure in the consciousness network and the simulation results show that due to the community structure, the awareness getting probability of the individual in the same community increases which leads to the increasing of the epidemic threshold.

Through the simulation analysis of the interaction between the two layers, we found that when the degree of unconsciousness of infectious disease is small and the probability of recovery of infectious disease increases, the infectious disease is not easy to break out. Therefore, we can through a variety of way to make the individual be aware of the disease and enhance the awareness of infectious disease exchange between individuals, so that reduce the incidence of infectious diseases.

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