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Probability analysis for estimation of annual extreme rainfall of Naogaon, Bangladesh

M. A. B. Barkotulla*

Department of Crop Science and Technology, University of Rajshahi, Rajshahi, Bangladesh

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Abstract

The daily rainfall data of 39 years of Naogaon rainfall station in Bangladesh from the year of 1971 to 2009 were analyzed to determine the annual one day maximum rainfall. The return period values were calculated by Weibull's plotting position and expected values were estimated by the best fit probability distribution. Three statistical goodness of fit test (Kolmogorove–Smirnov test (K-S), Anderson Darling test (A^2) and Chi-square test χ^2) were used to select the best fit probability distribution on the basis of minimum value of test statistic. Five probability distribution functions (*viz.* Gamma, Normal, Log-normal, Log-Pearson type-III and Gumbel distribution) were tested with the observed values to determine the best fit probability distribution. The Log-Pearson type-III distribution was the best fit probability distribution to estimate annual one day maximum rainfall for different return periods. The results would be helpful for agricultural scientists and engineers to take appropriate policy for agricultural development and constructions of small soil and water conservation structures, irrigation and drainage systems in the study area.

***Corresponding Author:** M. A. B. Barkotulla ✉ barkotru@yahoo.com

Introduction

Analysis of rainfall data strongly depends on its distribution pattern. It has long been a topic of interest in the fields of Agricultural Statistics in establishing a probability distribution that provides a good fit to daily rainfall data. Several studies have been conducted on rainfall analysis and the best fit probability distribution functions such as gamma, normal, log-normal, log-pearson type-iii and gumbel distributions were identified.

The parameters of the best fitted frequency distribution are estimated and the model is verified, then the model results could be considered dependable and can be used for variety of applications in water resources systems, planning and design. Many researches had been conducted to find the probability distribution function parameters of rainfall data. The normal, log-normal, gamma, weibull and pearson type distributions were used to fit the daily rainfalls in Pantagar, India (Sharma and Singh, 2010). The tests of goodness of fit were used the Kolmogorov-Smirnov, Chi-square and Anderson-Darling tests. They concluded that different types of distributions could fit for different locations.

Naogaon rainfall station having annual rainfall data of thirty nine (39) years were selected to perform this analysis. Results showed that the log-pearson type iii distribution performed the best fitted distribution. The normal, log-normal and gamma distributions have been used to model the maximum daily rainfalls. The results found that the log-normal frequency distribution was the best fit for both the one day annual maximum and 2 to 5 days maximum rainfall (Kwaku and Duke, 2007).

Rainfall is the principal phenomenon driving many hydrological extremes such as floods, droughts, landslides; its analysis and modeling are typical problems in applied hydrometeorology (Barkotulla, 2010). It is generally recommended that 2 to 100 years is sufficient return period for soil and water conservation measures, dam construction, irrigation and drainage works (Bhakar *et al.,* 2006).

For designing of various moisture conservation measures, rainfall analysis have been done for maximum one-day rainfall with different probability distributions. The observed values have been calculated by Weibull's formula.

The time series of historic rainfall data are characterized by their average and standard deviation, these values cannot be used to estimate design rainfall depths that can be expected with a specific probability or return period (Dirk, 2013).

In the present study, all the parameters have been used to describe the variability of rainfall and determined the statistical parameters and annual one day maximum rainfall at various probability levels using five probability distribution functions, *viz.*, gamma, normal, log-normal, log-Pearson type-III and Gumbel distribution. Estimation of rainfall depths (X_T) is the amount of rain that can be expected for a specific probability during a specific reference period are required for the management and design of irrigation and drainage projects.

Materials and methods

Data collection

Daily rainfall data of Naogaon rainfall station in Bangladesh was used for the present investigation. Time series rainfall records for the period of 39 years (1973 to 2011) were collected from Water Development Board, Bangladesh. Annual maximum daily rainfall was determined from these data (Table 1) and using statistical techniques for data analysis.

Statistical analysis

The statistical behavior of any hydrological series was described on the basis of certain parameters. The computation of statistical parameters includes mean, standard deviation.

Coefficient of variation and coefficient of skew ness were calculated as measures of variability of one day annual maximum daily rainfall. All the parameters were used to describe the variability of rainfall in the study.

Table 1. Annual and one day maximum rainfall (mm) for the period of 1971 to 2009.

Year	Annual rainfall	One day max. Rainfall	Year	Annual rainfall	One day max. Rainfall	Year	Annual rainfall	One day max. Rainfall
1971	1748.7	73.7	1984	1933	110.7	1997	1182.7	106
1972	985.8	104.1	1985	1499.6	157.5	1998	1729.4	97
1973	2077.9	177.8	1986	1629.8	278.1	1999	492.1	109.5
1974	1534.5	177.8	1987	2026.5	100.1	2000	1839.8	127.7
1975	1108.9	108	1988	2063.8	85.3	2001	1290.3	78
1976	1518.6	145.1	1989	1326.2	137.2	2002	1570.8	135
1977	1807.9	119.6	1990	1700.1	142.4	2003	1319.9	126.5
1978	1742.9	251.5	1991	1673.1	105	2004	1381.2	135
1979	1506	113	1992	1207	84	2005	1481.3	92
1980	1804.4	194.3	1993	2060.2	167	2006	1155.3	75
1981	1762.7	156.7	1994	1135.2	107	2007	1514	215
1982	1296.1	109.2	1995	1936.1	218.4	2008	1811	80
1983	1686.3	218.4	1996	1036	66	2009	1355	150

Methodology

Three well-known tests of goodness of fit namely Kolmogorove–Smirnov (K-S), Anderson Darling (A^2) and Chi-square (χ^2) tests were applied to the data series for investigating the fit of probability distributions. If the calculated statistic is lower than that critical value, it concluded that the distribution fits the data appropriately and the distribution can be accepted with the view to calculate recurrence intervals. The return period (also called the recurrence interval) T is the period expressed in number of years in which the annual observation is expected to return. Return period was calculated by Weibull's plotting position formula (Chow, 1964) by arranging one day maximum daily rainfall in descending order giving their respective rank as:

$$T = \frac{N+1}{R} \qquad (1)$$

Where, N = the total number of years and R= the rank of observed rainfall values arranged in descending order. The above 20% dependable rainfall (P = 0.20) has a return period of (1/0.20) = 5 years or on average once every 5 years rainfall depth is exceeded.

For designing of various soil and water conservation structures suitable for the study area, maximum one-day rainfall data have been predicted for different return periods based on the probability distribution analysis using frequency factors.

The expected rainfall values have been estimated by using frequency factors through the procedure described as under for different probability distributions (Table 2) i.e. normal, log-normal, gamma, log-pearson type iii and gumbel distribution (Chow, 1951).

Table 2. Different probability distribution function and their parameters.

Distribution	Probability density function	Range	Parameters in terms of the sample moment
Gumma	$f(x) = \frac{x^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{\left(\frac{-x}{\beta}\right)}$	$0 < x < \infty$	$\beta = \frac{S^2{}_x}{\bar{x}}$, $\alpha = \left(\frac{\bar{x}}{S_x}\right)^2$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$-\infty < x < \infty$	$\mu = \bar{x}$, $\sigma = S_x$,
Log-normal	$f(y) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2}$	$0 < x < \infty$	$\mu_y = \bar{y}$, $\sigma_y = S_y$
Log-Pearson Type III	$f(x) = \frac{1}{a_y\Gamma(b)}\left(\frac{y-c}{a}\right) e^{-\frac{1}{2}\left(\frac{y-c}{a}\right)}$	$0 < x < \infty$	$\mu = \bar{x}$, $\sigma = S_x$,
Gumbell	$f(x) = \frac{1}{\alpha}\exp\left[-\frac{x-\mu}{\alpha} - \exp\left(-\frac{x-\mu}{\alpha}\right)\right]$	$\infty < x < \infty$	$\mu = \bar{x} + 0.5772\alpha$, , $\alpha = \frac{S_x\sqrt{6}}{\pi}$

For Normal distribution, expected values of maximum one-day rainfall were estimated by the given formula (Chow, 1988).

$$K_T = \frac{X_T - \mu}{\sigma} \quad (2)$$

Where: X_T = expected maximum one-day rainfall corresponding to return period T; μ=mean of maximum one-day rainfall; σ= standard deviation and the frequency factor K_T for normal distribution is equal to z .

$$z = w - \left[\frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3}\right] \quad (3)$$

The value of z corresponding to an exceedence of p (p = 1/T) was calculated by finding the value of an intermediate variable w as per given formula.

$$w = \left[\ln\left(\frac{1}{p^2}\right)\right]^{1/2} \quad 0 < p \leq 0.5 \quad (4)$$

1-p is substituted for p when p > 0.5. In this case, the value of z is given a negative sign.

For Log-normal distribution, expected value of maximum one-day rainfall was estimated by the given formula.

$$Y_T = \mu + \sigma K_T \quad (5)$$

Where $Y_T = \ln(X_T)$; μ= mean and σ=standard deviation of Y_T; K_T = frequency factor corresponding to natural logarithm value.

In Log-Pearson type-iii distribution, the value of X (rainfall) was transformed to logarithm (base 10).

The expected value of rainfall X_T could be obtained by the following formula

$$X_T = \text{Antilog}\, X \quad (6)$$

and $\log X = \mu + \sigma K_T$ (7)

where, μ, σ, and C_s are the mean, standard deviation and coefficient of skew ness of logarithmic values of observed rainfall respectively. Frequency factor K_T is taken by the formula (Benson, 1968).

$$K_T = \frac{2}{C_s}\left[\left\{\left(z - \frac{C_s}{6}\right)\frac{C_s}{6} + 1\right\}^2 - 1\right] \quad (8)$$

In Gumbel distribution, the expected rainfall X_T was computed by the following formula

$$X_T = \mu + \sigma K_T \quad (9)$$

Where, μ is mean of the observed rainfall, σ is the standard deviation; K_T- frequency factor which is calculated by the formula (Gumbel, 1958).

$$K_T = -\frac{\sqrt{6}}{\pi}\left[0.5772 + \ln\left\{\ln\left(\frac{T}{T-1}\right)\right\}\right] \quad (10)$$

In case of gamma distribution, the frequency analysis was carried out the method as described by Haan, 1994.

Results and discussion

Different statistical parameters such as mean, standard deviation, co-efficient of variation, kurtosis and co-efficient of skewness of one day maximum and annual rainfall were described in Table 3. The distribution pattern of rainfall would be played an important role for crop planning and water conservation activities.

Table 3. Statistical parameters of one day maximum and annual rainfall from 1971 to 2009.

Parameter	Mean	S. D.	C. V.	Skewness	Kurtosis	Max	Min
One day max.	134.2	51	0.38	1.06	0.72	278.1	66
Annual	1536.7	348.3	0.22	-0.62	0.61	2077.9	492.1

Average annual rainfall was computed as 1536.7 mm and monsoon season was 1244.4 mm. Fig. 1 showed the trend analysis of annual rainfall of Naogaon rainfall station. In Fig. 2, 81% rainfall during monsoon season (June-September) 17% during pre kharif season and few rainfall 3% during rabi season were calculated for this area. The crop planning for major growing crops during Kharif season was carried out to get optimum crop production from the available rainfall.

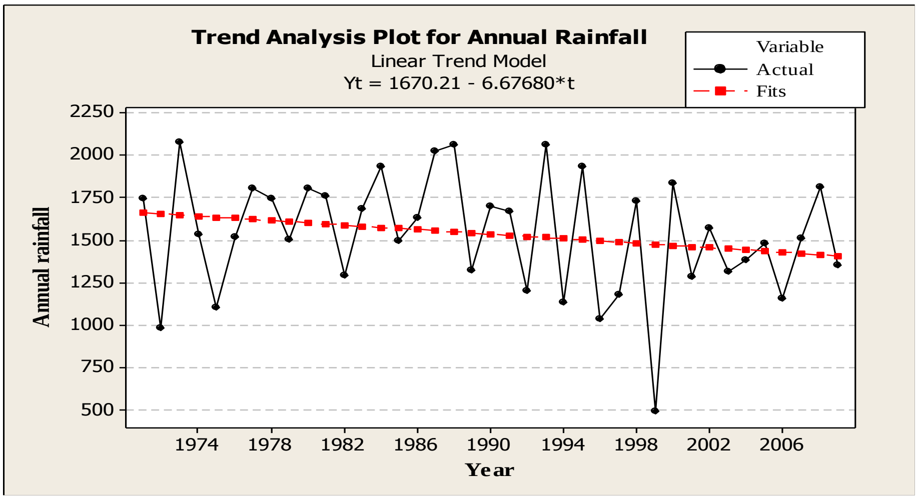

Fig. 1. The trend analysis of annual rainfall of Naogaon rainfall station.

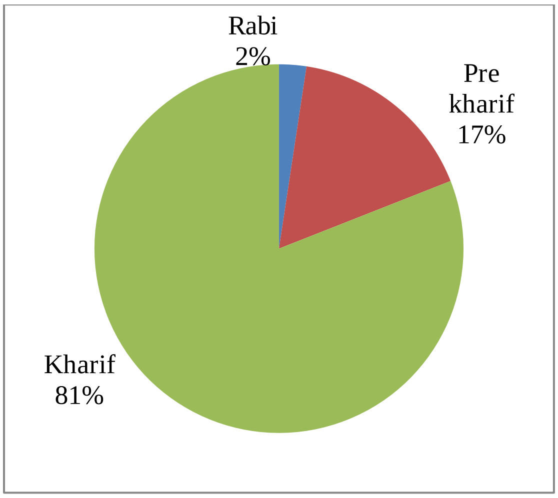

Fig. 2. Distribution of seasonal rainfall of Naogaon.

The Kolmogorove–Smirnov (K-S), Anderson Darling (A^2) and Chi-square (χ^2) test statistic for one day annual maximum rainfall were computed for five probability distributions.

The probability distributions with their test statistic were presented in Table 4. It observed that log-pearson type-iii obtained the first rank for maximum one day annual rainfall and it was identified as the best fit based on these three tests independently. The parameters of these distributions for one day annual maximum rainfall were estimated and the model was verified, then the model results could be considered dependable and could be used for variety of applications in water resources systems planning and design (Table 5).

Table 4. Selection the Best fit probability distribution using goodness of fit tests.

Distribution	Kolmogorove–Smirnov		Anderson-Darling		Chi-square	
Gamma	Statistic	Rank	Statistic	Rank	Statistic	Rank
Gumbel	0.10947	4	0.4451	4	3.1988	4
Log-Pearson type-iii	0.10328	2	0.28971	2	1.8225	3
Log-normal	0.08643	1	0.1913	1	0.49485	1
Normal	0.1075	3	0.30109	3	1.8208	2

Table 5. Parameters of the fitted probability distribution.

Distribution	One day Parameters
Gamma	α=6.926 β=19.379
Gumbel	σ=39.765 μ=111.27
Log-Pearson type-iii	α=35.483 β=0.05997 γ=2.7076
Lognormal	σ=0.35259 μ=4.8354
Normal	σ=51.001 μ=134.22

Based on the best fitted probability distribution, one year return period, the minimum rainfall 69.19mm in a day could be expected to occur with 97.5 per cent probability and 100 year return period, the maximum 315.44 mm rainfall could be expected with one per cent probability. Also the expected values of maximum one-day rainfall were 123.41, 168.76, 201.13, 279.21, 315.44 and 353.75 corresponding to 2, 5, 10, 50, 100 and 200 years returns periods respectively in Table 6. The results would be useful for agricultural development and constructions of small soil and water conservation structures, irrigation and drainage systems in the area.

Table 6. Expected one day annual maximum rainfall for various return periods.

Return period	Probability	Expected rainfall
1.02	97.5	69.194
1.05	95	74.499
1.11	90	81.736
1.25	80	93.202
2	50	123.41
4	25	158.22
5	20	168.76
10	10	201.13
20	5	233.94
40	2.5	267.93
50	2	279.21
100	1	315.44
200	0.5	353.75

Conclusion

Five probability distributions such as normal, log-normal, gamma, log-pearson type- iii and gumbel distributions were tested to determine the best fit probability distribution that described the annual one day maximum rainfall by comparing with Chi-square, Kolmogorov-Smirnov and Anderson-Darling goodness of fit tests. The results found that the log pearson type- iii distribution was the best fit probability distribution for annual one day maximum rainfall. Various statistical analysis of rainfall data are most appropriate for crop planning of rainfed crops while maximum one-day rainfall analysis becomes useful for the designing of various water conservation structures. Probability distribution helps to relate the magnitude of extreme hydrologic events like floods, droughts and water logging with their number of occurrences such that their chance of occurrence with time can be predicted successfully. The parameters would be support the crop to meet their water requirement timely during monsoon season. In order to provide supplementary irrigation, sufficient amount of water must be stored.

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