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Study of dispersion of drug in blood flow with the impact of chemical reaction through stenosed artery

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Abstract

The present study mathematically examines the dispersion of drugs in blood flow through cosine and sine-shaped stenosed arteries with the impact of chemical reaction where blood has been considered as Herschel-Bulkley blood fluid model. The deposition of cholesterols, fats and lipids plaques on the artery wall causes stenosis and leads to the narrowing of the artery. The stenosis shape and chemical reaction have an imperative impact on the effectiveness of the drug dispersion. The nonlinear differential equations are solved to achieve blood velocity. The dispersion function is gained in this model to generalized dispersion expression. Results show a good agreement with the results without the chemical reaction or stenosis.

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Introduction

Atherosclerosis, also known as stenosis, is a narrowing of the artery's diameter caused by the build-up of cholesterol, fat, and other harmful in the artery wall. A thickening of the stenosis could stop the flow of blood to tissues and organs and less oxygen enters the heart muscles, resulting the serious diseases and causes of death worldwide such as heart attack, stroke and high blood pressure. It is observed that stenosis affects the blood flow behaviour in a stenosed artery [Damiano and Bardin (2004), Gill *et al.,* (1970), Rane and Murthy (2016), (2017)]. The injection of the drug into the vein is one of the treatments to treat atherosclerosis and also to decrease cholesterol levels. In addition, the dispersion of the drug disturbs the blood flow behaviour. At low concentrations, the drug injection becomes therapeutic and the drugs can be toxic at high concentrations. Therefore, it is crucial to investigate the solute dispersion of blood flow to analyse the effectiveness of the drug dose and whether the dosage of the drug dose can be maintained within the target range and can be avoided the adverse toxic effect. It was the pioneer who investigated the drug dispersion in a straight tube for Newtonian fluid [Fry (1968), Young and Tsai (1973), Young (1968), Whitemore (1968), Shah (2021)]. It was explained by [Biswas and Laskar (2011), Echarm (1965), Deheri *et al.,* (2011)] that occurs the velocity movement combined with the molecular diffusion over the cross-section. Ali *et al.*, (2015), Petrila and Tarif (2005), Kumar and Shah (2022) investigated that Taylor's dispersion theory was only beneficial for a large time and thus [Naduvinamani *et al.,* (2004), Forrester and Young (1970)] improved the dispersion theory by adding the axial molecular diffusion effect.

Many researchers [Oka (1981), Pustěovská (2010), Kumar and Shah (2021)] examined the diffusion solute numerically and empirically for radial and axial molecular diffusion. Ratchagar and Vijaya Kumar (2019), Kumar and Shah (2020) investigated the solute dispersion process with the help of the expansion of a series of generalized dispersion model (GDM) models. The solute transportation in blood flow, like transferring drugs and toxins in the physical system, was examined and very significant in a few researches. It is found that the previous researchers only investigated the blood flow and solute dispersion in the fluid. Thurston (1976) and Dash *et al.,* (2000) investigated the influence of interphase mass transfer on the solute transportation in unsteady blood of Casson fluid for small arteries in order to gain a greater knowledge of solute dispersion in the non-Newtonian fluid. [Naduvinamani Savitramma (2013)] investigated the effect of yield stress on blood flow with the transportation of solute in a pipe.

[Roy and Beg (2021)] investigated transportation of solute by taking fluid as Herschel-Bulkley fluid with the unsteady solute dispersion using GDM in the channel. [Kumar and Raghavendra (2015), Chakravarty and Datta (1992), Shah (2021)], examined the transportation of solute for unsteady convective-diffusion equation in Poiseuille flow with wall absorption using GDM. The previous research only analysed the solute dispersion and did not consider the effect of chemical reaction and stenosis through cosine and sine-shaped stenosed arteries. In addition, it is found that the comparison of cosine and sine-shaped stenoses and the chemical reaction effect on the solute that disperses in the bloodstream have not been done. Therefore, as per the author's knowledge, research analyses the effect of stenosis and chemical reaction on solute dispersion in the flow of blood through cosine, sine-shaped stenoses. The velocity of blood is gained by solving the equations of momentum and constitutive analytically. The convective-diffusion equation has been analytically solved using the obtained velocity, and by following [Lee and Fung (1970), Bazrov (2010)], GDM has been implemented to obtain the solution of dispersion function and concentration of blood through cosine, sine-shaped stenosed arteries. The significance of the research will be beneficial to understanding the knowledge of drug injection physiological processes in the blood flow, predicting the drug dispersion effectiveness and predicting the appropriate drug dosage when the chemical reaction exists in the cosine and sine-shaped stenosed arteries.

Materials and Methods

Mathematical formulation

The blood flow in a circular pipe in the axial direction is recognized as laminar, axisymmetric, viscous incompressible and fully formed unidirectional flow, and H-B fluid is used to treat the blood. The geometry is shown in (Fig. 1) (Ku,1997).

The equations for flow are given by,

$$(d\bar{p}/dz) = -\frac{1}{r}\left(\frac{d\bar{r}\bar{\tau}}{dz}\right) \quad (1)$$

$$(d\bar{p}/d\bar{r}) = 0 \quad (2)$$

where,

R_0 is radius of the pipe,

$\bar{\delta}$ is height of the stenosis

$\bar{d}$ is the location of stenosis

$\bar{u}$ is velocity of fluid,

$\bar{z}$ and $\bar{r}$ are coordinates in axial and radial directions

$\bar{L}$ is artery length

$\bar{l}_0$ is the stenosis length

$\bar{R}(\bar{z})$ is the stenotic artery radius [Jaafar *et al.*, (2016)].

Fluid model equation can be written as,

$$\frac{d\bar{u}}{d\bar{r}} = -\frac{1}{\bar{\eta}_H}\left(\bar{\tau} - \bar{\tau}_y\right)^n \text{ if } \bar{\tau} > \bar{\tau}_y, \quad (3)$$

where $\bar{\eta}_H$ is the viscosity,

$$\bar{u} = 0 \text{ at } \bar{r} = \bar{R}(\bar{z}), \quad (4)$$

$$\bar{\tau} \text{ at } \bar{r} = 0, \quad (5)$$

Eqs. (1) and are given,

where, the radius of cosine-shaped stenosed artery is given by [Kudenatti *et al.*, (2013)] as follows:

$$\bar{R}(\bar{z}) = \begin{cases} R_0 - \frac{\bar{\delta}}{2}\left[1 + \cos\left(\frac{2\pi}{\bar{l}_0}\left(\overline{(z)} - \bar{d} - ((l_0/2))\right)\right)\right], \bar{d} \geq \bar{z} \geq \overline{(d} + \overline{l_0)} \\ R_0, otherwise \end{cases} \quad (6)$$

and the sine-shaped stenosed artery is given by

$$\bar{R}(\bar{z}) = \begin{cases} R_0 - \frac{\bar{\delta}}{2}\left[1 + \cos\left(\frac{2\pi}{\bar{l}_0}\left(\overline{(z)} - \bar{d} - ((l_0/2))\right)\right)\right], \bar{d} \geq \bar{z} \geq \overline{(d} + \overline{l_0)} \\ R_0, otherwise \end{cases} \quad (7)$$

The convective-diffusion equation with chemical reaction effect,

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{C}}{\partial \bar{z}.} = \overline{Dm}\left[\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial}{\partial \bar{r}}\right) + \frac{\partial^2}{\partial z^2}\right]\bar{C} - \bar{\beta}\bar{C}, \quad (8)$$

where $\bar{C}$ represent concentration of solute, $\bar{t}$ denotes time variable, $\bar{\beta}$ denotes chemical reaction rate. Eq. is given by

$$\bar{C}(\bar{r}, \bar{z}, 0) = \begin{cases} \bar{C}_0, & if|\bar{z}| \leq (\bar{z}_s/2) \\ 0 & if|\bar{z}| \leq (\bar{z}_s/2) \end{cases} \quad (9)$$

$$C(\bar{r}, \infty, \bar{t}) = 0, \quad (10)$$

$$(\partial\bar{C}/\partial r)(0, \bar{z}, \bar{t}) = 0 = (\partial\bar{C}/\partial r) \quad (11)$$

Non-dimensional variables for momentum and constitutive equations are given by,

$$u = \bar{u}/\bar{u}_0, \quad u_+ = \bar{u}_+/\bar{u}_0, \quad u_- = \bar{u}_-/\bar{u}_0, \quad u_m = \bar{u}_m/\bar{u}_0, \quad r = \bar{r}/R_0, \quad r_p = \bar{r}_p/R_0$$
$$\tau = \bar{\tau}/(\overline{\eta_H}\,\overline{u_0})/R_0{}^{1/n}, \tau = \bar{\tau}_y/(\overline{\eta_H}\,\overline{u_0})/R_0{}^{1/n}, R(z) = \frac{\bar{R}(\bar{z})}{R_0}, \delta = \frac{\bar{\delta}}{R_0}, d = \frac{\bar{d}}{R_0}, l_0 = \frac{\bar{l}_0}{R_0} \quad (12)$$

where

$$\overline{u_0} = \frac{\bar{R}(\bar{z})^{n+1}}{(n+1)\bar{\eta}}\left(-\frac{1}{2}\frac{d\bar{p}}{d\bar{z}}\right)^n \quad (3)$$

u velocity,

u_+ and u_- are the velocity of non-plug, plug

r is radial distance,

u_m is average velocity,

d is the location of stenosis,

δ is the stenosis height,

and l_0 is stenosis length in non-dimensional form.

The non-dimensional variables for convective-diffusion equations are given by

$$C = \frac{\bar{C}}{\bar{C}_0}, t = \frac{\bar{D}_m\bar{t}}{R_0^2}, Pe = \frac{\bar{R}(\bar{z})\bar{u}_0}{\bar{D}_m}, \beta = \sqrt{\frac{R_0^2\bar{R}}{\bar{D}_m}},$$
$$z_* = \frac{\bar{D}_m\bar{z}_*}{R_0^2\bar{u}_0}, z_s = \frac{\bar{D}_m\bar{z}_s}{R_0^2\bar{u}_0}, \quad (4)$$

where

t denotes the time,

Pe is Peclet number,

β is chemical reaction,

Z_* denotes the solute longitudinal distance and

Z_s denotes the solute length in non-dimensional form.

Non-dimensional scheme, Eqs. (1)
become

$$\frac{dp}{dz} = -\frac{1}{r}\frac{d}{dr}(r\tau), \quad (15)$$

$$\frac{dp}{dz} = 0, \quad (16)$$

$$-\frac{du}{dz} = (\tau - \tau_y)^n, if\tau > \tau_y, \quad (17)$$

$$\tau \text{ at } r = 0, \quad (5)$$

$$u = 0 \text{ at } r = R(z), \quad (6)$$

$$R(z) = \begin{cases} 1 - \frac{\delta}{2}\left[1 + \cos\left(\frac{2\pi}{l_0}(z - d - (l_0/2))\right)\right], d \geq z \geq d + l_0 \\ 1, \quad otherwise \end{cases} \quad (7)$$

$$R(z) = \begin{cases} 1 - \delta \sin\left[\frac{\pi(z-d)}{l_0}\right], & d \leq z \leq d + l_0, \\ 1, & \text{otherwise.} \end{cases} \quad (21)$$

By applying the non-dimensional variables of Eq. (4) into Eq. yields

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial z} = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{Pe^2}\frac{\partial^2}{\partial z^2}\right]C - \beta^2 C, \quad (8)$$

where

$$Pe = \frac{au_0}{D_m}$$

Applying the non-dimensional variables of Eq. (4) [17], into Eqs. given

$$C(r, z, 0) = \begin{cases} 1, if|z_.| \leq (z_s/2) \\ 0if|z_.| > (z_s/2) \end{cases} \quad (23)$$

$$(C(r, z_., 0) = C(r, \infty, t) = 0,) \quad (24)$$

$$(\partial C/\partial r)(0, z, t) = 0. = (\partial C/\partial r))(R(z), z, t), \quad (25)$$

Solution of the problem

Integrating Eq.
with respect to r forms the shear stress and it is given by $\tau_y = 2r_p$. Substituting $\tau = 2r$ and $\tau_y = 2r_p$ into Eq. and expand it into binomial series approximation in two terms.

Next, integrating the equation with respect to r subject to Eq. (6),

$$u_+(r) = \left(1 - \left(\frac{r^{n+1}}{R(z)^{n+1}}\right)\right) - (n+1)\left(\frac{r_p}{R(z)}\right)\left(1 - \left(\frac{r^n}{R(z)^n}\right)\right) + (n(n+1)/2)\left(\frac{r_p^2}{R(z)^2}\right)\left(1 - \left(\frac{r^{n-1}}{R(z)^{n-1}}\right)\right). \quad (26)$$

and by replacing $r = r_p$ into Eq. (26), the velocity in the plug flow region is given by,

$$u_-(r_p) = 1 - (n+1)\left(\frac{r_p}{R(z)}\right) + \left(\frac{n(n+1)}{2}\right)\left(\frac{r_p^2}{R(z)^2}\right) - \frac{n(n-1)}{2}\left(\frac{r^{n-1}}{R(z)^{n-1}}\right) \quad (27)$$

The mean velocity of blood flow for H-B is as follows:

$$u_m = \frac{(n+1)}{(n+3)}\left[1 - \frac{n(n+3)}{(n+2)}\frac{r_p}{R(z)} + \frac{n(n+3)(n-1)}{2(n+1)}\frac{r_p^2}{R^2(z)} - \frac{(n^4+2n^3-5n^2-6n+4)}{2(n+1)(n+2)}\frac{r_p^{n+3}}{R^{n+3}(z)}\right] \quad (28)$$

$$z_1 = z_* - u_m t.$$

Applying the approach Eq (8) can be obtained in term in derivative $\partial^j C_m / \partial z_1^j$,

$$C(r, z_1, t) = C_m(z_1, t) + \sum_{j=1}^{\infty} f_j(r, t)\frac{\partial^j C_m(z_1, t)}{\partial z_1^j} \quad (29)$$

where C_m denotes the concentration and given by

$$C_m(z_1, t) = 2\int_0^{R(z)} C(r, z_1, t)rdr \quad (30)$$

$$\frac{\partial C_m}{\partial t} + (u - u_m)\frac{\partial C_m}{\partial z_1} - \frac{1}{Pe^2}\frac{\partial^2 C_m}{\partial z_1^2}C_m\beta^2 + \sum_{j=1}^{\infty}\left[\left(\frac{\partial f_j}{\partial t} - L^2 f_j\right)\frac{\partial^j C_m}{\partial z_1^j} + (u - u_m)f_j\frac{\partial^{j+1} C_m}{\partial z_1^{j+1}} - \frac{1}{Pe^2}f_j\frac{\partial^{j+2} C_m}{\partial z_1^{j+2}} + f_j\frac{\partial^{j+1} C_m}{\partial t\partial z_1^j} + \beta^2 f_j\frac{\partial^j C_m}{\partial z_1^j}\right] = 0 \quad (31)$$

Now by modified dispersion model that consists of the dispersion coefficient [Misra *et al.*, (2006)].

$$\frac{\partial C_m}{\partial t} = \sum_{i=1}^{\infty} K_i(t)\frac{\partial^i C_m}{\partial z_1^i}(z_1, t) - \beta^2 C_m(z_1, t), \quad (32)$$

where $K_1(t)$ and $K_2(t)$ are the longitudinal convection and longitudinal diffusion or called the effective axial diffusivity, respectively. As $K_3(t)$ is very small for which, the terms of $K_3(t)$, $K_4(t)$ and others have been neglected. Using Eq. into Eq. (8) and equating the equation,

$$\left[{}^{\partial f_1}/_{\partial t} - L^2 f_1 + K_1(t) + u + u_m + \beta^2 f_1\right]\left({}^{\partial C_m}/_{\partial z_1}\right) + \left[{}^{\partial f_2}/_{\partial t} - L^2 f_2 + (u-u_m)f_1 + K_1(t)f_1 + K_2(t) - \frac{1}{Pe^2} + \beta^2 f_2\right]\left({}^{\partial^2 C_m}/_{\partial z_1^2}\right) + \sum_{j=1}^{\infty}\left[\frac{\partial f_{j+2}}{\partial t} - (u-u_m)f_{j+1} + L^2 f_{j+2} + \beta^2 f_{j+2} - \frac{1}{Pe^2} f_j + \sum_{i=1}^{j+1} K_i(t) f_{j+2-i} + K_{j+2}(t)\right]\frac{\partial^{j+2} C_m}{\partial z_1^{j+2}} = 0 \quad (33)$$

$$(\partial f_1/\partial t) - L^2 f_1 + (u-u_m)f_1 + K_1(t) + \beta^2 f_1 = 0 \quad (34)$$

$$(\partial f_2/\partial t) - L^2 f_2 + ((u-u_m)f_1 + K_1(t))f_1 + K_2(t) - (1/Pe^2) + \beta^2 f_2 = 0 \quad (35)$$

$$\left(\frac{\partial f_{j+2}}{\partial t}\right) - L^{j+2} f_2 + (u - u_m + K_1(t))f_{j+1} + \left(K_2(t) - \left(\frac{1}{Pe^2}\right)\right) f_j - \beta^2 f_{j+2} \sum_{i=2}^{j+1} K_i(t) f_{j+2-i} 0. \quad (36)$$

Now by using,

$$f_j(r,0) = 0,$$

(37)

$$\frac{\partial f_j}{\partial r}(0,t) = 0 = \frac{\partial f_j}{\partial r}\left[R(z),t\right]. \quad (38)$$

putting Eq. we get,

$$\int_0^{R(z)} f_j \; r \; dr = 0. \quad (39)$$

The dispersion function $f_1(r,t)$ plays a significant part in determining the variation of $C(r,z_1,t)$.

Expression of $f_1(r,t)$ is stated as

$$f_1(r,t) = f_{1s}(r) + f_{1t}(r,t), \quad (40)$$

putting Eq. into Eq. forms

$$\frac{\partial f_{1s}}{\partial t} + \frac{\partial f_{1t}}{\partial t} - L^2(f_{1s} + f_{1t}) + (u-u_m) + \beta^2(f_{1s} + f_{1t}) = 0 \quad (41)$$

Grouping coefficients of $f_{1s}(r)$ and $f_{1t}(r,t)$ written as,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_{1s}}{\partial r}\right) + (u-u_m) + \beta^2(f_{1s}) = 0 \quad (42)$$

$$\frac{\partial f_{1t}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_{1t}}{\partial r}\right) - \beta^2 f_{1t}. \quad (43)$$

Substituting Eq. into Eqs. and gives,

$$f_{1t}(r,0) = -f_{1s}(r), \quad (44)$$

$$\frac{df_{1s}}{dr}(r=0) = 0 = \frac{df_{1s}}{dr}(r=R(z)), \quad (45)$$

$$\partial f_{1t}(0,t)/\partial r = 0 = \partial f_{1t}(R(z),t)/\partial r \quad (46)$$

Using, Eq. (40) in the solvability condition (44) yields

$$\int_0^{R(z)} f_{1t} \; r \; dr = -\int_0^{R(z)} f_{1s} \, r \; dr = 0. \quad (47)$$

Eq. (42) and (45) the steady dispersion function $f_{1s\_}(r)$ with chemical reaction in the region of plug flow $0 \le r \le r_p$ is as follows:

$$f_{1s}(r) = -\frac{1}{\beta^2}\left(\frac{2}{n+3} + \frac{(2(n+1))r_p}{(n+2)R(z)}\right) + \left(\frac{n r_p^2}{R(z)^2}\right) - \left(\frac{n(n-1)r_p^{n+1}}{2R^{(n+1)}}\right) + \left(\frac{n^4+2n^3-5n^3-6n+5}{2(n+1)(n+3)R(z)^{(n+3)}}\right) + C. \quad (48)$$

The function $f_{1s+}(r)$ is very long, thus, the expression is not indicated in this paper.

Solving Eq. (43) with the conditions and and solvability condition (48), the

$$f_{1t}(r,t) = e^{-\beta^2 t}\sum_{m=1}^{\infty} A_m \, e^{\lambda_m^2} J_0(\lambda_m r) \quad (49)$$

$$A_m = \frac{\int_0^{R(z)} J_0(\lambda_m r) f_{1s}(r) \, r \, dr}{\int_0^{R(z)} J_0^2(\lambda_m r) \, r \, dr} = \frac{2}{J_0^2(\lambda_m)}(I_1 + I_2). \quad (50)$$

$$I_1 = \int_0^{r_p} J_0(\lambda_m r) f_{1s-}(r) r dr, \quad (51)$$

$$I_2 = \int_{r_p}^{R(z)} J_0(\lambda_m r) f_{1s-}(r) r dr, \quad (52)$$

Eqs. (51) and (52) have been calculated with the help of Matlab software for obtaining unsteady dispersion function.

Results and discussion

This research aims to analyse the effects of stenosis and chemical reaction on dispersion coefficient in blood flow. The parameters involved in this research are the stenosis height, stenosis length, stenosis location, plug core radius, yield stress, power-law index and chemical reaction.

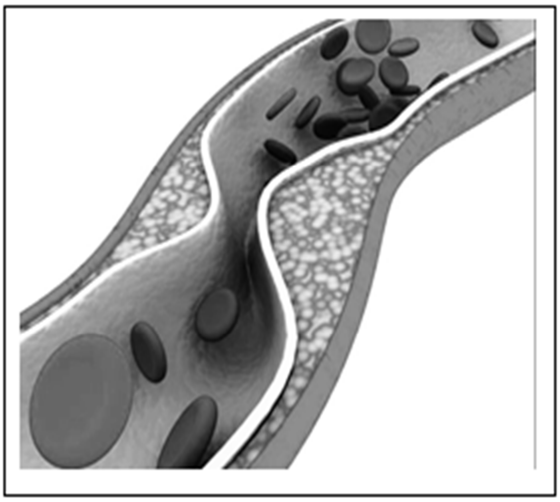

Fig. 1. Blood flow in a stenosed artery.

The following is the range of parameter values used in this research: plug core radius r_p : 0 to 0.2 power-law index n, chemical reaction rate β : 0-8 (Miah *et al.,* 2020, Sadique and Shah, 2022). The dispersion function f_{1s} , f_{1t} for steady and unsteady dispersion function, the stenosis did not exist with the effect of chemical reaction for both cosine and sine-shaped stenosed arteries are validated in Fig. and Fig., respectively $t = 0.1$ and $r_p = 0.1$. [5, 8, 23] examined the solute dispersion in blood flow without the stenosis.

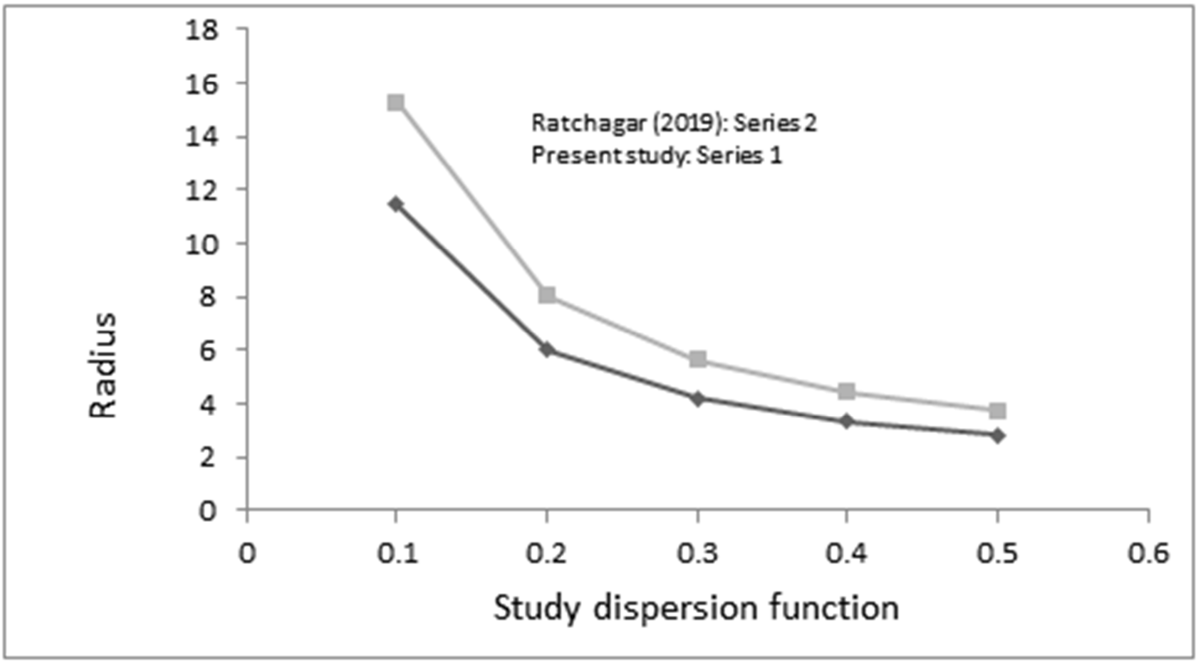

Fig. 2. Validation of steady dispersion function f_{1s} with radius r when $\delta = 0,\ l_0 = 3,\ d = 2,\ z = 4,\ R(z) = 1,$, and [Chakravarty and Datta (1989), Ellahi *et al,*. (2014)].

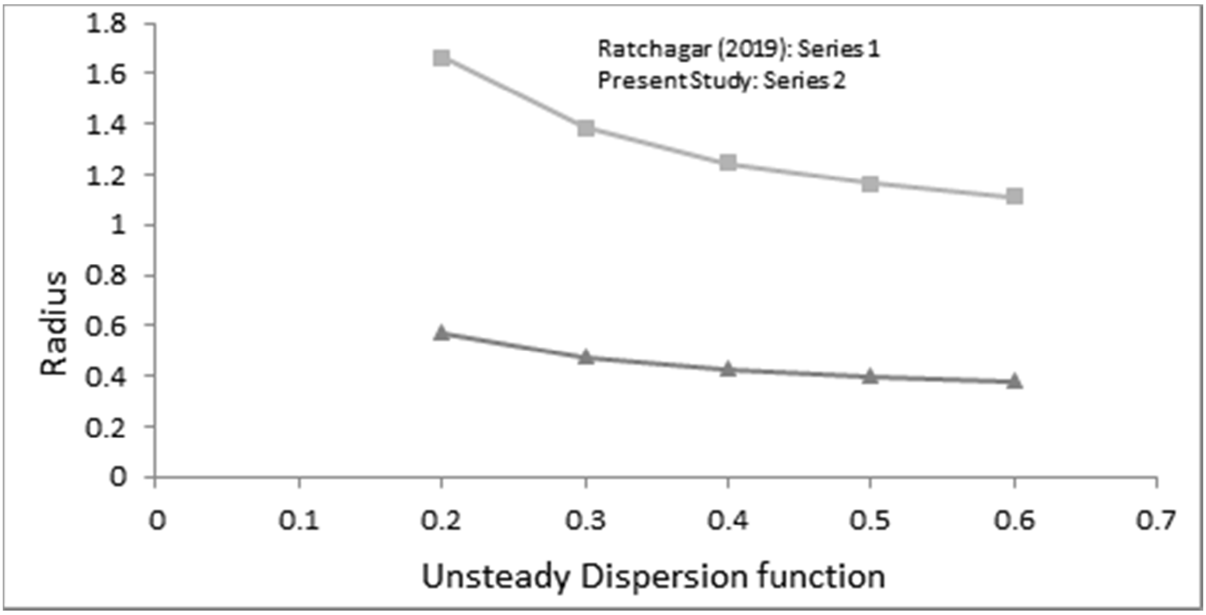

Fig. 3. Validation of unsteady dispersion function f_{1t} with radius r when $\delta = 0$, $l_0 = 3$, $d = 2$, $z = 4$, $R(z) = 1$, $\beta = 0.5$, $t = 0.1$, $n = 0.95$ and $r_p = 0.1$. [Jaafar *et al.*, (2016 & 2017)].

The results of the steady and unsteady dispersion functions for both stenosis shapes when the stenosis height is absent $(\delta = 0)$ exhibit similarities with the findings of Chakravarty (1987).

Conclusion

The steady and unsteady dispersion of a solute in Herschel-Bulkley blood model has analyzed with the existence of stenosis and chemical reaction. In this study, a comparison between the cosine and sine-shaped stenosed arteries has been made. The results without stenosis have been compared with existing literature. The analysis shows that the dispersion coefficients are affected by the stenosis shape. This study is very important and theoretically implies in the biomedical industry to predict the drug dispersion changes in blood flow and design some medical devices.

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