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Teaching through whole-classroom discussions as an efficient tool for improving mathematical reasoning abilities of students

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Abstract

Our purpose of conducting this research is to study the impact of promoting whole-class discussions among students on their mathematical reasoning abilities. In this manuscript, we report an experiment within teaching through whole-class discussions by using open-ended problems in classroom provided an efficient tool for improving students' reasoning abilities. A school was randomly selected among high schools and two classes were randomly selected among grade 8th classes of this school. These classes were assigned into one of two teaching methods: whole-class discussion (the experimental group, 27 students) and traditional instruction (the control group, 30 students). Students in both groups were instructed same topics and learning (materials formal booklet of 8th grade) by the same experienced teacher. Analysis of covariance showed a significant interactive and positive effect of experimental method on improving students reasoning abilities.

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Introduction

Reasoning stands at the center of mathematics learning. Mathematics relies on logic and it is through this logic that mathematics knowledge can be justified. Without reasoning, mathematicians would not be able to convince other people that their conclusions are true, and make sense (Muller & Maher, 2009). Reasoning can be defined as the process of drawing conclusions on the basis of evidence or stated assumptions (Martin & Kasmer, 2009). Mathematical reasoning refers to thinking through mathematics problems logically in order to arrive at solutions (Selden & Selden, 2003). Reasoning in mathematics is often understood to mean formal reasoning, in which conclusions are logically deduced from assumptions and definitions. We reason when we examine patterns and detect regularities, make conjectures, and evaluate or construct sound deductive arguments. These activities are fundamental to making sense of content. Many concepts and processes, such as generalization, can help students gain insights into the nature and beauty of mathematics (NCTM 2000, p. 15). Students need to develop increased abilities in justifying claims, proving conjectures, and using symbols in reasoning. They can be expected to learn to provide carefully reasoned arguments in support of their claims.

According to Yopp (2010), there are two forms of the reasoning. These are *inductive* reasoning and *deductive* reasoning. Inductive reasoning is a logical process in which a learner proceeds from particular evidence to a conclusion, which is viewed as true (Johnston, 2002). In other words, inductive reasoning is generally used to prove or establish that a given statement is true for some natural numbers. An example of inductive reasoning is empirical reasoning. In empirical reasoning, the learner uses a particular case to generalize for all cases. Deductive reasoning is a logical process whereby something that is already known and everyone agrees that it is true, is applied to a particular case (Johnston, 2002). Communication has been a central theme in the reform of mathematics classrooms due to its role in facilitating learning through discourse (Cazden 2001, Knuth & Peressini 2001). Brandt and Tatsis (2009) argue that teaching of mathematics must focus on encouraging collective mathematics argumentation and support learners to express their reasoning. It is very crucial that learners are encouraged to verbalize their ideas and thoughts. In other words, the way learners are involved in explaining, reasoning and justifying content related actions in the mathematics classroom is crucial for their learning. In the process of argumentation, explaining and reasoning in the mathematics classroom, learners are encouraged to communicate their mathematics reasoning. However, Brandt and Tatsis (2009) do not give suggestions on how learners should be encouraged to communicate their mathematics reasoning.

In mathematically productive classroom environments, students should expect to explain and justify their conclusions (NCTM, 2000, p. 342). Engaging in effective collaborative interaction requires all members of the classroom community to be active and critically constructive participants. For many students this means a shift away from the more traditional role of passive receivers of instruction. The pedagogical intent is that students are involved in learning communities in which all participants have opportunities to engage in productive mathematical discourse (Manoucheri & St. John, 2006). Thompson and Chappell (2007) recommended embedding reading, writing, speaking, and listening of mathematics into every class.

In whole-class discussions, the teacher is in charge of the class, just as in direct instruction. However, in this talk format, the teacher is not primarily engaged in delivering information or quizzing. Rather, he or she is attempting to get students to share their thinking, explain the steps in their reasoning, and build on one another's contributions. These whole-class discussions give students the chance to engage in sustained reasoning. The teacher facilitates and guides quite actively, but does not focus on providing answers directly. Instead, the focus is on the students' thinking (Chapin, O'Connor and Anderson 2003, p.17). Blanton and Stylianou (2014) found that transactive reasoning which defined as criticisms, explanations, justifications, clarifications, and elaborations of one's own or another's ideas, to be a useful construct for analyzing whole-class discourse. They suggest that classroom discourse that helps students' appropriate transactive reasoning can support their learning of proof. They have stated how an instructtional practice that promoted transactive reasoning supported students in developing a habit of interacttion based on critiquing, clarifying, justifying, explaining, and elaborating their mathematical ideas.

However, there are more-immediate reasons for emphasizing mathematical communication in school mathematics. Interacting with others offers opportunities for exchanging and reflecting on ideas; hence, communication is a fundamental element of mathematics learning (NCTM 2000, p. 348). Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills (Yackel & Hanna, 2003). In a very recent article, the potential of critical mathematics (CM) was investigated in terms of its ability to disrupt traditional patterns of student participation in classroom discourse. Analyses of transcripts of whole-class discussions showed that engaging reformist critical mathematics (RCM) activities featured higher levels of elaborate student engagement with and lower levels of resistance to whole-class discussions than those that dissipated participation (Brantlinger, 2014).

To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks (NCTM, 2000, p. 17). Mathematical reasoning can be promoted by high level demand procedural tasks that seek to enhance understanding and sense-making in learners as they explore relationships and mathematical conceptual understandding and processes (Stein *et al.*, 1996). The high level task questions maybe open ended or explanatory questions which require learners to formulate a way of solving them without relying on already known procedures and calculations (Kilpatrick *et al.,* 2001). The kinds of experiences teachers provide clearly play a major role in determining the extent and quality of students' learning. Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge (NCTM, 2000, p.21). Osana, Lacroix, Tucker and Desrosiers (2006) stress the use of open-ended tasks which favour students' involvement in class activities and encourage them to explore and investigate, increase their motivation for generalization, look for models and links, communicate, discuss and identify alternatives. Viseu and Oliveira (2012) have conducted an experiment in teaching the topic 'Sequences and Regularities' with open-ended tasks, using a qualitative and interpretative approach. Data were collected during two class observations, from two interviews and by analyzing the activities of the students. An exploratory task was chosen in the first lesson and an investigative one in the second. One month separated the two lessons, and during this time the teacher read and discussed texts on mathematics communication. Observation of the first lesson showed that the communication in the classroom was mostly focused on the teacher, which provided little student-student and student-class interaction. In the second observed lesson, the teacher changed the attention she paid to what each student said and did, encouraging the students to ask each other and encouraged student-class and the student-student communication (Viseu and Oliveira, 2012).

Although Osana *et al.* (2006) consider that tasks of an open nature stimulate students to engage in class activities, Nicol (1999) stresses the importance of teachers knowing how to listen to their students in order to encourage them to discuss the classroom activities. Clark and co-workers presented four strategies for mathematical communication in detail: (1) posing rich tasks, (2) creating a safe environment, (3) asking students to explain and justify solutions, and (4) actively processing one another's ideas (Clark, Jacobs, Pittman and Borko, 2005). Chapin, O'Connor and Anderson (2003) represented five productive talk moves: (1) Revoicing, (2) Asking students to restate someone else's reasoning (3) Asking students to apply their own reasoning to someone else's reasoning (4) Prompting students for further participation (5) Using wait time. A key challenge that mathematics teachers face in enacting current reforms is to orchestrate whole-class discussions that use students' responses to instructional tasks in ways that advance the mathematical learning of the whole class (Ball, 1993; Lampert, 2001). Stein, Engle, Smith and Hughes (2008) have presented a model and set out five practices for whole-discussion facilitation:(1) anticipating the students' likely answers to cognitively demanding mathematical tasks; (2) monitoring the students' answers to the tasks during the exploratory phase; (3) selecting some students to present their mathematical responses during the discussion phase; (4) intentionally sequencing the students' responses; and (5) helping the class to make mathematical connections between the students' different responses.

Although previous studies have emphasized on the relation between classroom discussions and mathematical reasoning ability (NCTM, 2000, Chapin, O'Connor & Anderson, 2003, Blanton & Stylianou, 2014,Yackel & Hanna, 2003, Yankelewitz, Mueller & Maher, 2010), however, to the best of our knowledge, there isn't any quantitative researches investigating impact of classroom discussions on the mathematical reasoning ability. Therefore conducting a research in this context seems necessary to acknowledge teachers of mathematics.

The aim of the current study is to investigate and comparing impact of using whole-class discussions and traditional teaching methods the students' mathematical reasoning abilities, quantitatively.

Materials and methods

Procedure

In this research, participants were 57 students. A school was randomly selected among high schools (Tehran, Iran), and two classes were randomly selected among grade 8th classes of this school. These classes were assigned into one of instruction methods: whole-class discussion (the experimental group, N=27) and traditional instruction (the control group, N=30). Students of both groups were exposed to the same topics and learning materials and were taught by the same experienced teacher.

For facilitating whole-class discussions, the teacher used five practices according a model that presented by Stein, Engle, Smith and Hughes (2008). In this research, the teacher posed cognitively demanding mathematical tasks (open-ended problems) for the effective use of student responses in whole-class discussions. This study was performed during 3 months, 4 hours per week.

In the experimental group, the teacher posed open-ended problems in the classroom, then facilitates and guides quite actively, but does not focus on providing answers directly. For providing a contributive and productive whole-class discussion, the teacher gave the students opportunities to discuss mathematical tasks with one another, present solution strategies, or help each other to develop solutions and appropriate problem solving strategies. The teacher was attempting to get students to share their thinking, explain the steps in their reasoning, and build on one another's contributions. The control group was exposed to the traditional method of instruction, in which the teacher introduced the new concepts to the class, and then students practiced the problems relating to the new concepts. If any student could not solve a problem of booklet, the teacher solves it and explains the strategies and steps of solution.

Examinations

The pre-test and post-test mathematical reasoning ability assessment examinations were employed in the present study. Prior to the beginning of the study, and the end of the study, all students were administered the pre-test and the post-test, respectively. Each examination is constructed of 18 problems (selected from TIMSS questions) that examined students' reasoning ability to solve mathematical problems. Validity of the problems was confirmed by two specialists. Examination time was 75 minutes. The scores on each problem is 0-4. According to Miyazaki's model, four basic levels are distinguished and scored 1-4 (level A=4, B=3, C=2 and D=1), the blank problem is scored 0 (Miyazaki, 2000). Thus, the total scores ranged from 0–90. Alpha Cronbach reliability scores were 0.81, both on the pretest and posttest.

Analysis

A one-way between-groups analysis of covariance (ANCOVA) was conducted to compare the effectiveness of whole-class discussion designed to improve participants' reasoning abilities. The independent variable was the type of instruction (whole-class discussion and traditional instruction), and the dependent variable consisted of scores on reasoning test. Test administered after the instruction was completed. Participants' scores on the pretest administration of the reasoning ability were used as the covariate in this analysis.

Results and discussion

In this research, after selecting two classes as experimental and control groups, a pre-test (mathematical reasoning ability assessment examination) was taken from all students. Then, whole-class discussion and traditional teaching methods were employed for the experimental and control groups, respectively. At the end of the study (after 3 month), all students were administered the post-test, mathematical reasoning ability assessment examination.

Table 1 presents the mean scores of pre-test and post-test for students of two experimental and control groups and the corresponding standard deviations According to the results, no significant differences were found between the two groups on the pre-test mean scores prior to the beginning of the study. However, the mean scores of post-test was increased for the experimental group with respect to the corresponding mean scores of the control group.

Table 1. Students' mean scores and standard deviations on pre-test and post-test.

Group	test	N	Mean	Variance	SD
Experimental	Pre-test	27	47.78	159.795	12.64
	Post-test	27	71.81	153.93	12.41
	Pre-test	30	50.83	131.52	11.47
Control	Post-test	30	66.73	90.62	9.52

Before conducting an ANCOVA, we applied Levene test for evaluating ANCOVA assumptions. According to results of table 2, homogeneity of variances, [$F= 0.101$, $p = 0.752 >0.05$], was met.

Table 2. Results of Levene test.

	df_1	df_2	F (Levene)	P
Pre-test	1	55	.101	.752

According to the results of table 3, homogeneity of regression slopes was established. Also, preliminary checks were conducted to ensure that there was no violation of the assumptions of normality, linearity, and reliable measurement of the covariate. Considering amounts of $P= .052> .05$ and $F= 3.123$, we concluded that interaction between independent variable and pre-test was not significant, therefore the assumption of homogeneity of regression slope does hold.

Table 3. Tests of Between-Subjects Effects (Dependent Variable: posttest).

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	725.310[a]	2	362.655	3.123	.052
Intercept	10390.316	1	10390.316	89.464	.000
group * pretest	725.310	2	362.655	3.123	.052
Error	6271.567	54	116.140		
Total	279479.000	57			
Corrected Total	6996.877	56			

a. R Squared = .104 (Adjusted R Squared = .070).

Table 4 represents main outputs of ANCOVA. Considering table 4(row 3), p= .049<.05 and F= 4.044, linear relationship between covariate (pre-test) and the independent variable (teaching method) assumption does hold. There was not a strong relationship between the pretest and posttest scores on the reasoning ability test, as indicated by a partial eta squared value of 0.070 (Table 4).

Table 4. Tests of Between-Subjects Effects (Dependent Variable: post-test).

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	828.843[a]	2	414.421	3.628	.033	.118
Intercept	10212.427	1	10212.427	89.408	.000	.623
Pretest	461.906	1	461.906	4.044	.049	.070
Group	472.995	1	472.995	4.141	.047	.071
Error	6168.034	54	114.223			
Total	279479.000	57				
Corrected Total	6996.877	56				

a. R Squared = .118 (Adjusted R Squared = .086).

Also, considering table 4(row 4), p= .047<.05 and F= 4.141, was significant. So, after excluding the effect of covariate (pre-test), there is a significant difference between mean scores of two experimental and control on the post-test. This difference was attributed to teaching method as independent variable. Therefore, we conclude that mathematical reasoning abilities of students who were instructed through whole-classroom discussion were increased with respect to the students that were instructed by traditional method.

Conclusions

Communication and also reasoning and proof are two important standards among the five process standards emphasized by the National Council of Teachers of Mathematics (NCTM 2000). According to NCTM," Communication is an essential part of mathematics and mathematics education" (NCTM 2000, p. 60). The mathematics communication standard highlights the importance of communicating student's mathematical thinking to peers and teachers which necessary for effective learning. Although many researchers have examined some models of communication such as small-group discussion, talk or discourse to achieving a desired outcome of teaching and facilitating independent student thinking, helping strike an appropriate balance for effective learning and to engage, empower, and subjectify students, (Leikin and Dinur 2007, Tsay, Judd, Hauk, and Davis 2011, Brantlinger 2014), our study helps to understanding the relationship between two standards of NCTM, communication and reasoning ability in learning mathematics.

The results of this study show that promotion of whole-class discussions using open-ended problems may improve the students' mathematical reasoning abilities. This confirms whole-class discussions as an effective teaching method for improving students from inductive reasoning to higher levels of deductive reasoning. Increasing students' reasoning abilities may support their learning of proof (Blanton and Stylianou, 2014).We hope that the results of this research would help students and teachers to applying and engaging in whole-class discussions in mathematics classrooms.

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